Homework 1

Due Monday, January 21, 2013

Notes: Please email me your solutions for these problems (in order) as a single Word or PDF document. If you do a problem on paper by hand, please scan it in and paste it into the document (although I would prefer it typed!).

1. (5 pts) I would like to get to know you and your interests so that I can provide the best possible educational experience for you in this course. Please describe yourself – your general background, education, interests, and goals. What specifically would you be interested in learning about computer vision? Are there any application areas that you are particularly interested in? Are you currently doing thesis research that might benefit from computer vision? Also if possible, please paste in a photo of yourself so I can start to learn names!

2. (10 pts) Show (by hand) that $(AB)^T = B^T A^T$. Use as an example the matrices

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

3. (15 pts) Consider the symmetric matrix $\mathbf{C} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$. Calculate by hand
   a. The determinant of the matrix, $|C|$.
   b. The inverse of the matrix, C^{-1}.
   c. The trace of the matrix.
   d. The eigenvalues of the matrix.

4. (15 pts) Consider a rotation about the X axis of 1.1 radians, followed by a rotation about the Y axis of -0.5 radians, followed by a rotation about the Z axis by 0.1 radians (this order of rotations is called the "XYZ fixed angles" convention).
   a. Give the 3x3 rotation matrix corresponding to the rotations above.
   b. Since the rotation matrix is "orthonormal" matrix (i.e., a square matrix whose rows and columns are orthogonal unit vectors), its inverse is equal to its transpose. Show this.
   c. Give the 3x3 rotation matrix where the same rotations described in part (a) are done in the opposite order; i.e., first a rotation about the Z axis of by 0.1 radians, followed by a rotation about the Y axis of -0.5 radians, followed by a rotation about the X axis by 1.1 radians (this convention is called "ZYX fixed angles"). The matrix should be different.

5. (15 pts) Consider the "cameraman.tif" image in Matlab. Assume that the camera that took this image can be modeled by a pinhole camera, with square pixels, and the optical center is

at the center of the image. Assume that the tall building in the distance is 40 meters wide, and the distance to the building is 2 km. What is the focal length of the camera, and the field of view?

6. (15 pts) Using the parameters found in the previous problem, draw a "full moon" as a white circle in the "cameraman.tif" image[1]. The unit vector direction to the moon is (ux,uy,uz) = (0.0984, -0.1476, 0.9841) in camera coordinates, where we use the usual convention that +X is to the right, +Y is down, and +Z is forward. Give the code you used, and the image.

7. (25 pts) A camera observes the following 7 points, defined in WORLD coordinates (meters):

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  6.8158    7.8493    9.9579    8.8219    9.5890   13.2690   10.8082
-35.1954  -36.1723  -25.2799  -38.3767  -28.8402  -58.0988  -48.8146
 43.0640   43.7815   40.1151   46.6153   42.2858   59.1422   56.1475
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The pose of the camera with respect to the world is given by the following:

- Translation of camera origin with respect to the world is (10,-25,40) in meters.
- Orientation of the camera with respect to the world is given by the angles provided in problem #4.

(a) Compute the homogeneous transformation matrix, ${}^{C}_{W}H$, assuming that the convention being used is "XYZ fixed angles".
(b) Transform the 7 points from WORLD coordinates to CAMERA coordinates.
(c) Project the 7 points in CAMERA coordinates onto an image. Use the following parameters for the camera. Size of the image is 256 columns (width) by 170 rows (height). Center of projection is at the image center. Effective focal length is 400 pixels. Show the resulting image (hint: it should be a familiar object).

[1] You will have to look up the angular size of the moon as seen from earth.